PHYS 232 – Assignment #3

Due Friday, Mar. 1 @ 11:00

For all of these problems, to receive full credit, you must:

- Show all of your work.
- Explain your reasoning using words.
- Include large and neat diagrams when necessary.
- Present your solution in a neat a logical way that is easy to follow.

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1. On a certain kind of slot machine there are 10 different symbols that can appear in each of three windows. The machine pays off different amounts when either one, two, or three lemons appear.

(a) How many different possible outcomes are there each time you play this slot machine?

(b) Use the Binomial distribution to find the probability of getting exactly one lemon P_1, exactly two lemons P_2, and three lemons P_3.

(c) If this is a fair machine (no cut for the house), what should the slot machine pay out when you get exactly one lemon, exactly two lemons, and three lemons. Assume that it costs \$1 to play the machine. Let D_1, D_2, and D_3 denote the dollar amounts paid out when getting one, two, and three lemons respectively. Your payouts should satisfy the condition that the average payout $\langle D \rangle = \$1$. Also, assume that you want to pay 1/3 of the total money collected to people that get one lemon, 1/3 to people that get two lemons, and 1/3 to people that get three lemons.

2. In the limit that the number of trials n is large and that the mean $\mu = np \gg 1$, the Binomial and Gaussian distributions are approximately the same:

$$P_{\mathrm{B}}(x; n, p) \approx P_{\mathrm{G}}(x)\,\Delta x$$
$$\frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \approx \frac{\Delta x}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

Use the Gaussian distribution to calculate the approximate probability of getting 15 heads if you toss a fair coin 25 times. Calculate the exact probability using the binomial distribution.

3. A radioactive sample contains 5.0×10^{19} atoms, each of which has a probability $p = 3.0 \times 10^{-20}$ of decaying in any given five-second interval. (a) What is the expected average number μ of decays from the sample in five seconds? (b) Compute the probability $P_{\mathrm{P}}(x)$ of observing x decays in any five-second interval for $x = 0, 1, 2, 3$. (c) What is the probability of observing 4 or more decays in any five-second interval?

4. The average rate of disintegrations from a certain radioactive sample is known to be roughly 20 per minute. If you wanted to measure this rate to within 4% of its actual value, for approximately how long would you plan to count?